S-Transform based filter applied to the analysis of non-linear dynamic behaviour of soil and buildings

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ABSTRACT

One of the main tools used to study the dynamic response of systems is certainly the Fourier transform. This tool is very useful and reliable if one wants to investigate the response of a stationary system, i.e. a system that, doesn't changes its characteristics over time. Conversely, when one wants to study the evolution of the dynamic response of a system whose features vary with time, the Fourier transform is no longer reliable. In this regard, several mathematical tools were developed to study time-variable dynamic responses. Soil and buildings, subject to transient forcing such as an earthquake, may change their characteristics over time with initiation of non-linear phenomena. In most cases, both for soil and buildings, abandoning the linear elastic field could represent a problem. This paper proposes a new methodology to approach the study of non-stationary response of soil and buildings: a band-variable filter based on S-Transform. In fact, thanks to the possibility of change the bandwidth over time, it becomes possible to extract from a generic signal only the response of the system focusing on the variation of individual modes of vibration. The paper starts from examples and applications on synthetic signals, then examines possible applications to the study of the non-stationary response of soil and buildings.

Keywords: Dynamic Identification – Structural Monitoring – Filtering – S-Transform

1. INTRODUCTION

Generally, the study the evolution over time of dynamic characteristics of a system has been addressed in an approximate way trying to stationarizing it by using techniques that might not be appropriate. For example, one of the main tools used to analyse dynamical systems is the Fourier transform. This technique, like all tools which have their basis on the assumption of stationary behaviour of the system, appears to be not appropriate when one wants to conduct studies on systems that change over time. In this context the concept of change is understood as the time evolution of dynamic parameters that characterize the system under study. In fact, going to evaluate the Fourier Transform of a non-stationary signal, what ensues is a graph in the frequency domain which allows us to evaluate the average size of each spectral amplitude, assessed on the entire length of the analysed signal. It is immediately obvious that the transient phase of the signal, which appears to be a fraction of the whole signal, it is completely distorted. Another tool widely used to study the transients is the STFT (Short Time Fourier Transform) (Gabor, 1946). This technique exceeds certain limits of the simple Fourier transform, giving us some indications on the variation over time of the spectral characteristics of analysed signal. Unfortunately also this integral transformation has some limits which tend to distort the result. Indeed, the values of the moving-averaged spectral magnitudes have no absolute value, but may be used only for relative comparing. In order to assess the STFT it is necessary to choose the length of the time-window, this choice cannot be done arbitrarily, but is bound by the necessity of having a good resolution both in time and in frequencies domain. In order to have a good time resolution, we should choose a very short time-window, but in this case we would have a very low resolution in the frequency domain. On the contrary, in order to have a good frequency resolution, we should use a very long time-window. In the end, depending on the problem that one wants to address, it is necessary to do a suitable choice of the time-window length.
In recent decades, several techniques for time-frequency signals analysis have been developed. The most widely used are the Wavelet Transform and the Wigner-Ville Distribution. These other transformations present a number of advantages compared with the Fourier transform and the STFT, but do not allow a fair assessment of the local spectrum. In other words, these instruments are insufficient to a correct evaluation of the spectral characteristics taking into account the instantaneous variations. A tool that overcomes these limitations and allows us to accurately assess both the spectral characteristics and their local variation over time is the S transform (Stockwell *et al.*, 1996). This transformation, for a signal $h(t)$, is defined as:

$$S(\tau, f) = \frac{|f|}{2\pi} \int_{-\infty}^{+\infty} h(t) \cdot e^{-\frac{(\tau - t)^2 \cdot f^2}{2}} \cdot e^{-i \cdot 2\pi \cdot f \cdot t} dt \tag{1.1}$$

where t is time, f is frequency and τ is a parameter that controls the position of Gaussian window along the time axis. As it has been defined in the transformation, the properties of Gaussian window scale derive from the continuous wavelet. However, the condition of zero mean for the wavelets it is not satisfied. Furthermore, compared to the wavelet transform, the S-Transform changes the shape of the real and imaginary coefficients over time together with the temporal translation of the Gaussian window. On the contrary, the wavelet transform does not have this property: the entire waveform translates over time, but never changes its shape (Parolai, 2009). The S-Transform also has many important properties. For example, the S-Transform can be written as an operator that is function of the Fourier spectrum (Stockwell *et al.*, 1996):

$$S(\tau, f) = \frac{|f|}{2\pi} \int_{-\infty}^{+\infty} H(\alpha + f) \cdot e^{-\frac{2 \cdot \pi^2 \cdot \alpha^2}{f^2}} \cdot e^{-i \cdot 2\pi \cdot \alpha \cdot \tau} d\alpha \qquad \text{with } f \neq 0 \tag{1.2}$$

Furthermore, it can be shown that (Stockwell *et al.*, 1996):

$$H(f) = \int_{-\infty}^{+\infty} S(\tau, f)\, d\tau \tag{1.3}$$

So it is possible to completely recover the function *h(t)* by using the following relation (Stockwell *et al.*, 1996):

$$h(t) = \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} S(\tau, f)\, d\tau \right) \cdot e^{i \cdot 2\pi \cdot f \cdot t}\, df \tag{1.4}$$

The last, but not least important property is the linearity of the S-Transform (Stockwell *et al.*, 1996):

$$S\{data(t)\} = S\{signal(t)\} + S\{noise(t)\} \qquad \text{where } S \text{ stands for } \textit{S-Transform} \tag{1.5}$$

Thanks to this property it is possible to extract the processed information by the signal of interest, without altering its characteristics.

2. BAND-VARIABLE FILTER (S-filter)

In the past, many authors have tried to develop methods to clean up a generic recorded signal from noise (Douglas, 1997). Eqn. 1.5 suggested the possibility to filter the signals using Stockwell transform (Pinnegar and Eaton, 2003; Schimmel and Gallart, 2005; Ascari and Siahkoohi 2007; Simon *et al.*, 2007; Parolai, 2009). This paper discusses the possibility to use the S-filter to extract the dynamic characteristics of systems that evolve over time by acting simultaneously in both frequency and time domain. The filter was built using the properties of convolution, linearity and invertibility of the S-Transform. The algorithm on which the method for filtering is based can be summarized in a few steps:

1- Assessment of S-Transform $S(\tau, f)$ of the signal *h(t)*;

2- Generating the matrix filtering $G(\tau, f)$;

3- Calculating convolution in the frequency domain $M(\tau, f) = G(\tau, f) * S(\tau, f)$;

4- Assessment of the filtered signal $h_f(t)$ through the calculation of the inverse S-transform matrix $M(\tau, f)$.

$$h_f(t) = \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} [S(\tau, f) * G(\tau, f)]\, d\tau \right) \cdot e^{i \cdot 2\pi \cdot f \cdot t}\, df \tag{2.1}$$

The tapered-cosine window shown in Figure 1 was used to build the filtering matrix $G(\tau, f)$.

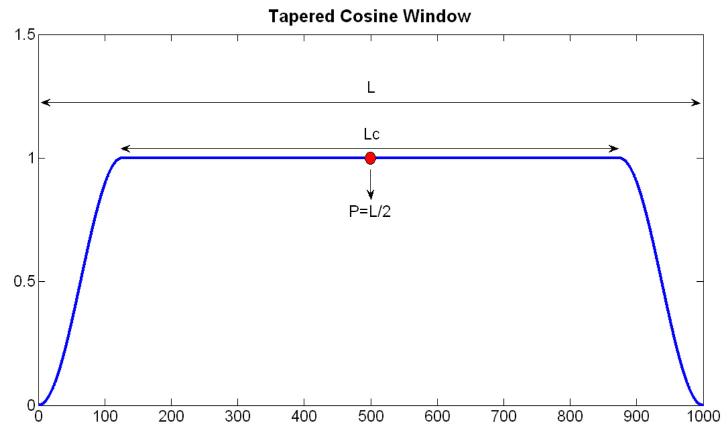


Figure 1. Example of tapered-cosine window used for build the filtering matrix.

This window identifies a generic $G(\tau_k, f)$. For the correct definition of the filtering window it is necessary to define the total width of the window (L) and the width of the constant section identified by Lc. In this work, after several tests a constant was chosen: r = $(L\text{-}Lc)/L$ = 0.25.
Moreover, it is necessary to define from time to time, the total width of the window. This width can be chosen bearing in mind that the number of points that make up a single window can be evaluated using the following relation:

$$Np(1Hz) = \frac{Lt}{fs} \tag{2.2}$$

where Lt is the signal length in time domain (expressed in percentage) and fs is the frequency sampling. In fact, if the window filtering has sizes smaller than $S(\tau, f)$ it is necessary to insert zeros up to the length (in frequency) of the matrix. The definition of the filtering matrix could be a time-demanding operation. In order to simplify this process, it is convenient to try to automate it. The implementation proposed defines the array selecting the local spectrum using a graphical user interface (GUI). The selection of points is done by clicking with the mouse cursor on the screen window where the time-frequency matrix is plotted. The cursor represents the midpoint P of the window filtering. The routine is designed so that the user can select a few points and the computer performs a linear interpolation. The algorithm was implemented in Matlab ©.

3. APPLICATION ON SYNTHETIC SIGNAL

In order to demonstrate the functionality of this tool several tests on different kinds of non-stationary signals have been performed. Let us start with a synthetic non-stationary signal, with a transient superimposed. Consider a synthetic non-stationary signal, that it is slowly changing frequency over time. The signal length is equal to 22 sec with 1 sec of zeros at beginning and end. Exceeded the zeros, the initial frequency is 11 Hz, in the central part reaches the minimum frequency, equal to 3 Hz, then return at a frequency equal to 11 Hz. The frequency variation follow a parabolic law. The signal is plotted in Figure 2a. This signal was summed with a signal with unitary amplitude, like the first one, and a constant frequency equal to 9 Hz.

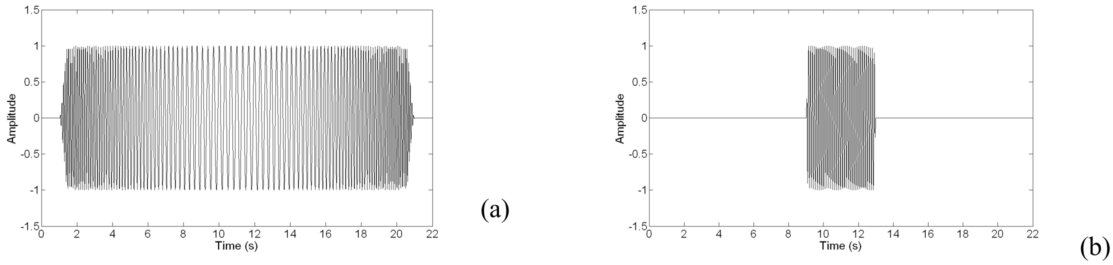


Figure 2. (a) Frequency variable sinusoid. (b) Constant amplitude sinusoid

The result is a non-stationary signal with frequency variable with a strong disturbance in the central part, corresponding to the minimum values of the frequency of the main signal. It should be noted that the frequency of the disturbance is contained within the range of variation of the main signal, and this could be an insurmountable problem when using a classical approach. Figure 3a reports the sum of the signals, while Figure 3b shows the Fourier spectrum of the same signal. It is evident from the spectrum shown in Figure 3b, that looking only at the Fourier spectrum it is impossible to distinguish the main signal from noise, and using a traditional filter, it would be impossible to eliminate the interference without affecting the original signal. It is in situations like this one, that a filter with variable band-pass should allow us to isolate the main signal from noise. To identify the different components from the signal, we evaluate the S-Transform. From Figure 4a it is evident the advantage of using the S-Transform in place of the classical Fourier transform. In fact, observing the figure enables us to clearly distinguish the two signals and the variation of the energy content over time.

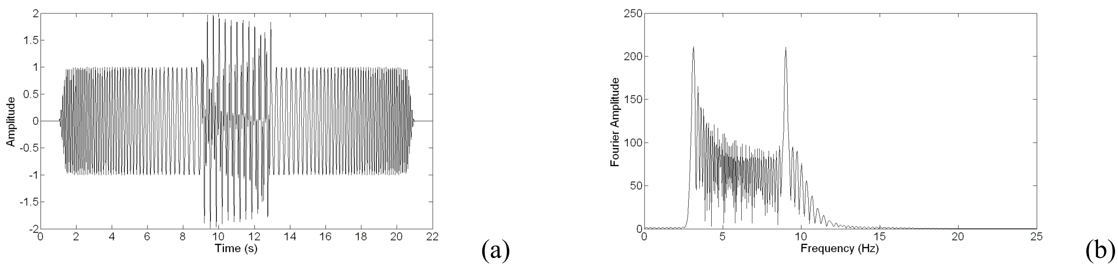


Figure 3. (a) Signal and noise. (b) Fourier spectra.

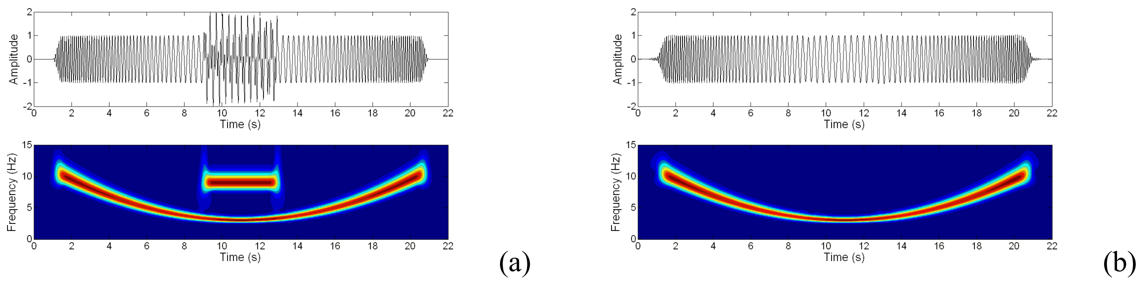


Figure 4. (a) S-Tansform of signal; (b) Filtered signal using band-variable filter.

As an example we show the filtering matrix used to eliminate the interference (Figure 5b). Figure 5a shows the GUI construction of a filtering matrix. The local spectrum is selected using the mouse cursor.

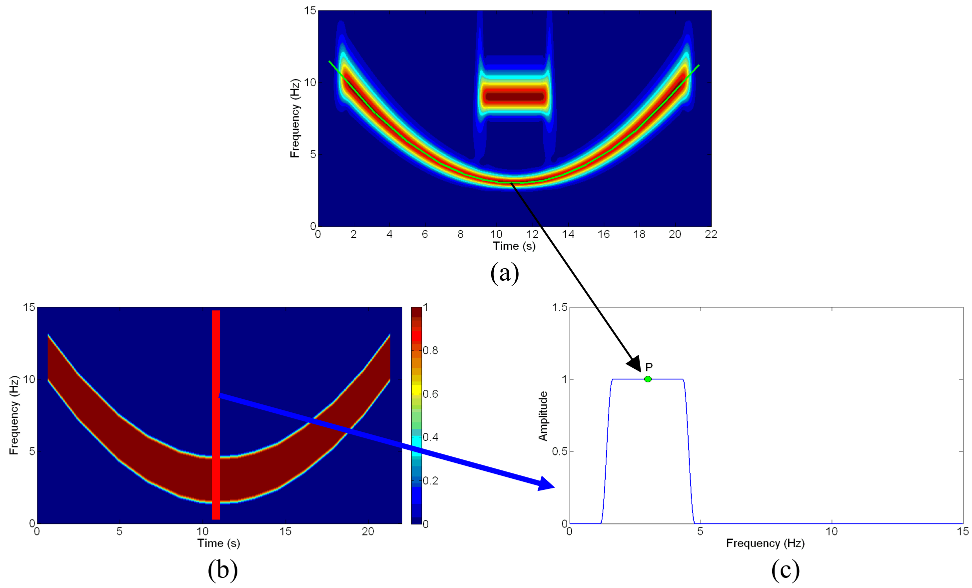


Figure 5. Example of filtering procedure.

The green curve represents the set of points P, central to the filtering window. The width of the window, as mentioned above, is chosen so as to be able to isolate the spectrum of interest, in this case, based on the information extracted from S-Transform, a width of 3.60 Hz was chosen, corresponding to a points number equal to

$$Np = \frac{Lt}{fs} \cdot 3.63 = 80 \tag{3.1}$$

Obviously, the remaining part of each local spectrum, is filled with zeros (Figure 5c). It is important to underline that the values used to construct the filter box are all real numbers, then the part of the complex matrix resulting from the calculation of the S-Transform is not altered. The result is a matrix filtering that changes the amplitude without altering the phases of the signal (Figure 5b). So, we get a variable band-pass filter that gives us a filtered signal with the phases corresponding to those of original signal. Similarly to what was done to extract the variable frequency signal, we may extract the component that we currently have called interference.

4. APPLICATION ON REAL DATA

To demonstrate how it is possible to pick from a real recording only the information of interest, we will show three real case histories: the first signal was recorded during an experimental campaign on a full-scale structures, the second one was recorded on a building while it was being damaged by an aftershock of the Molise earthquake (Italy) in 2002 and the third one was registered during the Kobe earthquake in 1995 in a site where soil suffered severe non-linearity leading to liquefaction. During the Bagnoli experiment (Gallipoli *et al.*, 2006), developed in order to test several shape-memory devices for retrofitting buildings, it was also possible to study the radiation effects of buildings on the free-field ground motion (Ditommaso *et al.*, 2010a). Several release tests were performed on the monitored building, thanks to a contrast frame structure, that was moved from the equilibrium condition and was left floating in free oscillations. The tests were numerous and in different conditions of degradation of the building. In order to monitor the behaviour of the structure in different conditions and equipped with various devices, several control devices have been installed on the building and on the surrounding area. Thanks to the presence of the accelerometers installed in the surrounding area it was possible to record the ground motion caused by the oscillating building. Figure 6a shows the displacements recorded at a station placed 10 m away from the building, the S-Transform for registration and a zoom in the frequency range 0-3 Hz where we find most of the energy released from the structure.

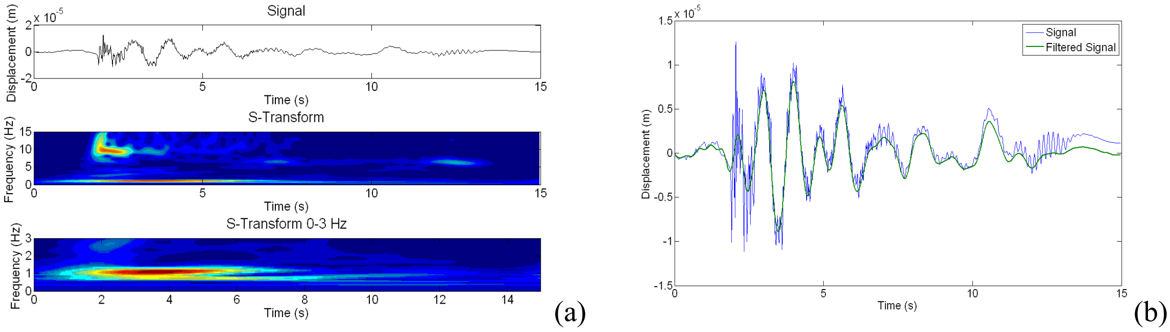


Figure 6. (a) Free-field recording at 10m from building; (b) Comparison between filtered and original signal

At this point, using the band-variable filter we try to isolate the principal mode of vibration from higher modes and noise content. In order to extract the local spectrum varying over time we set a total width for the window filtering equal to 1 Hz. The result (Figure 6b) shows that it is possible to extract from a real signal the local spectrum with higher energy content that is varying over time as expected due to the stiffening of the system caused by the shape-memory devices. Moreover, it is possible to avoid the phase distortions that can be introduced by standard causal filters.
The second example is taken form the seismic sequence in Molise (Italy). The town of Bonefro endured little damage (I_{MCS} = VII) except for two reinforced concrete buildings that have suffered serious damage. These buildings are very similar, both as a project as construction, and are located near to each other in an area where the surface layers appear to be formed by soft sediment. The main difference between the two buildings appears to be the height. The most damaged building has 4 floors, while the less damaged has three floors, then, should be more rigid. As a result of the earthquake of the first of November (M 5.3) the tallest building had an additional damage to structural parts. Fortuitously, a seismometer located within the building recorded the seismic response of the building during the progression of damage (Mucciarelli *et al.*, 2004). Obviously, the building has exhibited a strongly non-linear behaviour, leading to a 40% decrease of the frequency of the fundamental mode. Event recordings for the two orthogonal components in the horizontal plane are shown in Figure 7 with the respective S-Transform. It is clear that notwithstanding the saturation of the seismometer during the event, it is possible to follow the evolution of the seismic behaviour of the building in time-frequency domain. A detailed study, dealt with different methodologies, about the behaviour of this building and the possible effects of double resonance soil-structure was made by Mucciarelli *et al.* (2004).

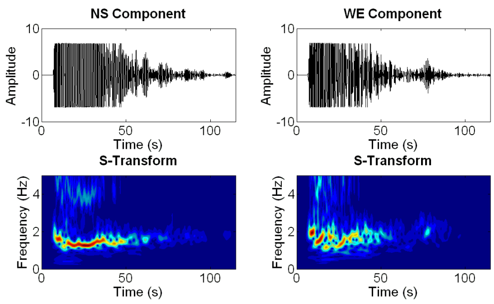


Figure 7. Velocimetric recordings within the building (Bonefro).

At this point we focus the attention on the NS component of the recorded motion of building. Using the S-Transform analysis is evident that the building starts from a frequency equal to 2.2 Hz, reaches the minimum frequency during the maximum excitation around 1.2 Hz before recovering in the final part of the signal at about 1.8 Hz. At this point we try to isolate the response of the building using the band-variable filter. Looking the S-Transform result for the signal, is clear that to isolate the building response is sufficient to take a width of the filtering window equal to 1 Hz. The result of filtering is shown in Figure 8. It is very interesting to see that, thanks to the selection of the local spectrum, the S-filter tool is able to reconstruct the dynamic response of the structure, with a negligible amount of noise even during non-stationary response. So, having at the disposal the recordings at all levels of the building, we could study the behaviour of individual modes of vibration and their variation over time. It is obvious that if, during the seismic response of a building we observe changes in the dynamic behaviour of individual modes of vibration, we are dealing with a structure which is changing its dynamic characteristics very likely due to damaging.

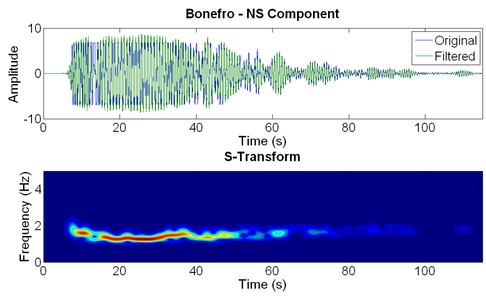
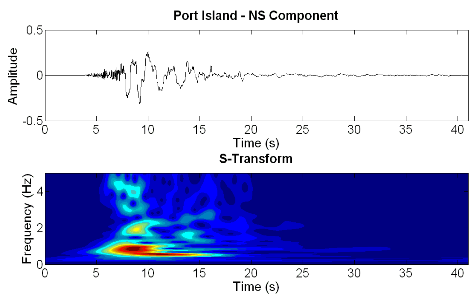
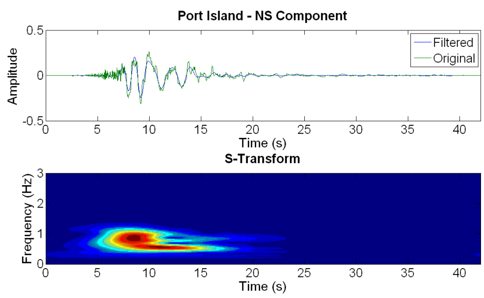


Figure 8. Extracting non-stationary behaviour of the main mode of the structure.

Finally, we apply the band-variable filter to extract the fundamental ground response during the Kobe earthquake (Japan) in 1995. The site of Port Island during the earthquake experienced soil liquefaction and thus a large variation of stiffness for the surface layers resulting in a variation of the main frequency of oscillation. Figure 10a reports the NS component of the recording for Kobe earthquake at Port Island site, acquired in free-field conditions. The signals recorded during this earthquake, have been extensively studied (Aguirre and Irikura, 1997) along with all the data acquired, even at different depths.



(a)



(b)

Figure 10. (a) NS Component of Kobe earthquake recording; (b) Band-variable filter application on Kobe earthquake recording (NS component).

Analysing the S-Transform of the signal is clear that most of the energy is concentrated in a single mode of vibration and it has a strongly frequency changes after just one cycle. Following the frequency change, which we know is due to liquefaction of the subsoil, we do not observe stiffness recovery, at least in this short period of time. We try now to extract information about the characteristics of the first mode of vibration of the soil, trying to remove from the registration all the noise and the response of potentials higher modes. For this purpose we used a filter with a window width equal to 1 Hz. S-Transform of the filtered signal and the comparison between the two signals is shown in Figure 10b. It is evident that also in this case the filter gives very interesting results.

5. CONCLUSIONS

People who deal with signal analysis and dynamic characterization of systems are in search of mathematical tools that enable them to study in detail the phase of non-stationary response of a dynamic systems. We have seen how classical techniques based on Fourier transform provide good results when the response of the system is stationary (Picozzi *et al.*, 2010), but completely fail when the system exhibits a non-linear behaviour. So that classical techniques fail, it is not necessary that the a building reaches damage: even the non-stationarity of the input and the possible interaction with the ground and/or adjacent structures can show the uselessness of classic techniques (Ditommaso *et al.*, 2010b). In 1996, Stockwell has introduced a new powerful tool for the signals analysis: the S-Transform. Compared with the classical techniques for the time-frequency analysis, this transformation shows a much better resolution and also offers a range of fundamental properties such as linearity and invertibility. By exploiting these properties, it was possible to developed a filter whose band varies both in time and in frequency domains, which is very useful to study the characteristics of non-linear signals. This tool becomes necessary if we want to isolate the response of individual modes of vibration, of soil and buildings, their dynamic characteristics evolving over time as a result of vibration due to seismic events. The ability to investigate the non-linear response of soils and buildings opens new scenarios, gives us the opportunity to explore new possibilities. For example, the ability to isolate individual modes of vibration of a building make possible to explore their variation over time, evaluating the change in modal curvature. It is known that this parameter is strongly linked to the building damage during a seismic event (Pandey *et al.*, 1991). Therefore, being able to evaluate the modal curvature during the maximum excursion in non-linear field and isolating it from superimposed signals, we grasp a better understanding of the mechanisms of damage and will also allow us a more precise location of the damage on the structure.

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