

# A FAST METHOD FOR STRUCTURAL HEALTH MONITORING OF STRATEGIC BUILDINGS

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## Abstract

The demand of a widespread health monitoring of strategic buildings in seismic areas has emphasized the need to realize studies in order to verify the feasibility of economic and fast procedures to detect anomalous vibrations on a large number of buildings, to perform post earthquake monitoring and to define first damage scenarios. Structural health monitoring systems are usually realized with a large number of sensors, suitably distributed on the structure, often involving complex elaborations of big amounts of data. When applied on a large number of buildings these systems can hardly result to be realizable for the necessary long time and the high cost to extract useful information. Within the Italian research RELUIS project, funded by the Italian Department of Civil Protection, a specific task deals with the possibility of applying a fast procedure to determine the damage evolution on a large number of structures after seismic events. The method developed and presented in this paper is based on a statistical approach that uses the most significant data recorded on the top floor of the building, with the purpose of extracting the value of the maximum inter-story drift expected along the building height, adopted as damage indicator. The parameters considered in the method are the maximum top acceleration, the modal frequencies variation and the equivalent structural viscous damping variation. Experimental tests carried out on scaled R/C models and several numerical non linear dynamic analyses have been considered to verify the feasibility of this approach.

## Introduction

The assessment of an increasing number of aged structures and infrastructures requires a huge effort, especially if the purpose is to provide a faithful evaluation of seismic risk. The current practice of periodic visual inspections, for the safety evaluation appears more and more inadequate. During the past two decades, a significant amount of researches (Ditommaso et al. 2010; Picozzi et al., 2010a; Picozzi et al., 2010b; Ponzo et al., 2010) have been carried out in the field of non-destructive damage evaluation (NDE) utilizing the changes in the dynamic response of a structure. The NDE methods can be classified into four levels [Stubbs et al. 2000], according to the specificity of the information provided by a given approach [Rytter 1993]: (i) Level I methods that only detect if damage has occurred; (ii) Level II methods that detect if damage has occurred and simultaneously determine the location of damage; (iii) Level III methods that identify if damage has occurred, determine the location of damage as well as estimate the severity of damage; (iv) Level IV methods that identify if damage has occurred, determine the location of damage, estimate the severity of damage, and evaluate the impact of damage on the structure. Each level of damage identification described above requires a gradual increasing amount of data and more complex algorithms, with a consequent rising of the costs, elaborations time and error probability.

A specific task of the RELUIS project 2005-08, funded by the Italian Civil Protection Department (DPC), deals with devising and implementing effective and fast procedures to get useful information on the damage evolution on a large number of strategic buildings during and after seismic events. The damage detection procedure developed within RELUIS project and described in this paper is a simplified 1<sup>st</sup> level procedure based on a statistical approach, to continuously check the safety and reliability of strategic buildings. The feasibility and the cost optimisation are the most important goals of the simplified procedure in order to favour a widespread diffusion of such systems. The method detects the damage evolution by comparing the dynamic response of the building before, during and after the earthquake. The procedure is designed in order to use only few sensors located on the top floor of the monitored building by which it extract some dynamic parameters (i.e. maximum top acceleration, modal frequencies variation and equivalent structural viscous damping variation). These parameters opportunely combined through a non linear relationship obtained by statistic evaluation, allow to evaluate the Damage Index. The main

assumptions at the base of the procedure and some experimental applications and numerical simulations provided to verify the effectiveness of the method are here described.

## 1 THEORETICAL APPROACH

Level I methods are usually based on the variation of vibration frequencies or damping. Such methods are often convenient because simple and robust. They employ a reduced number of sensors installed in the structure, although they can lead to wrong evaluations. In fact, the variation of vibration frequencies is not necessarily representative of damage evolution, but it can also be determined by a variation of the temperature or the masses and stiffness configuration, especially for deformable structures as reinforced concrete or steel frames [Doebeling e Farrar, 1998]. The proposed simplified method for structural health monitoring of strategic buildings starts from a reduced number of records acquired on top floor (velocity or acceleration) and overcomes some limitations of the Level I methods. Indeed, the method considers four parameters drawn from recorded signals, as defined in the following paragraph, and makes a combination of these parameters to estimate, through a non linear correlation analysis, the maximum inter-storey drift. Finally, the maximum value of the estimated drift along the building height can be assumed as correction factor of the Damage Index determined by classic first level's method.

### 1.1 Definition of Parameters

The maximum absolute top acceleration (MATA) represents the first instrumental parameter considered in this method. It can be evaluated by a suitably filtered signal recorded on the top floor of the building. An appropriate arrangement of recording sensors on the structure allows the reconstruction of all movements of the floor, rotation included. Other instrumental parameters considered by the method are: i) the percent variations ( $\Delta f_1$ ) between the fundamental frequency of the building before the seismic event  $f_{init}$  and the minimum one  $f_{min}$  corresponding to the maximum non linear behaviour of the building; ii) the percent variations ( $\Delta f_2$ ) between initial and final frequency, as given by equations 1 and 2.

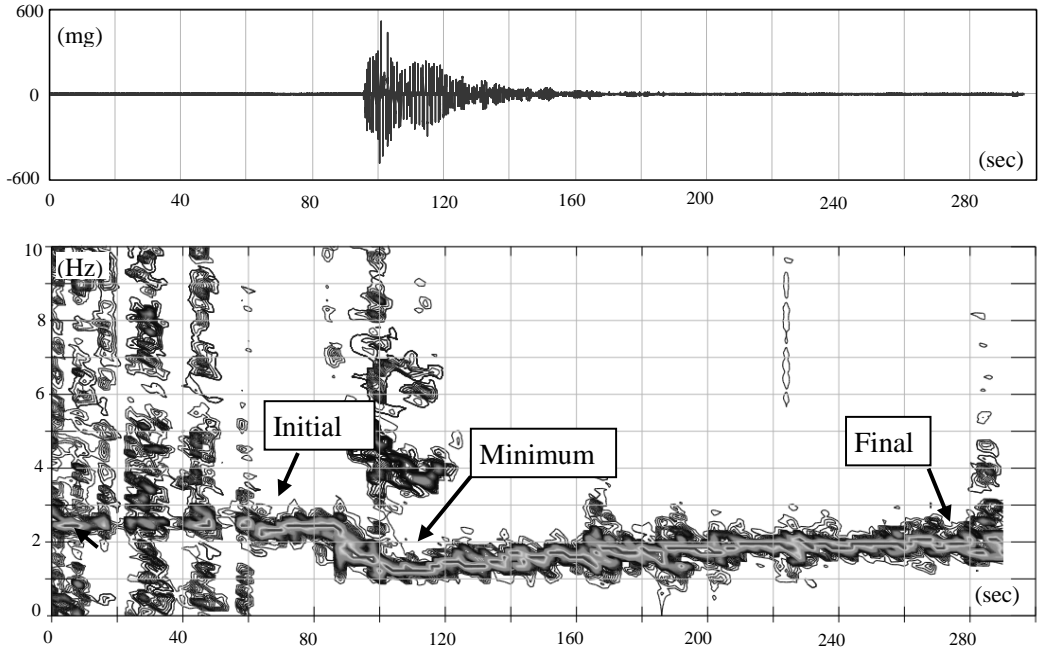
$$\Delta f_1 = (f_{init} - f_{min}) / f_{init} \quad (1)$$

$$\Delta f_2 = (f_{init} - f_{fin}) / f_{init} \quad (2)$$

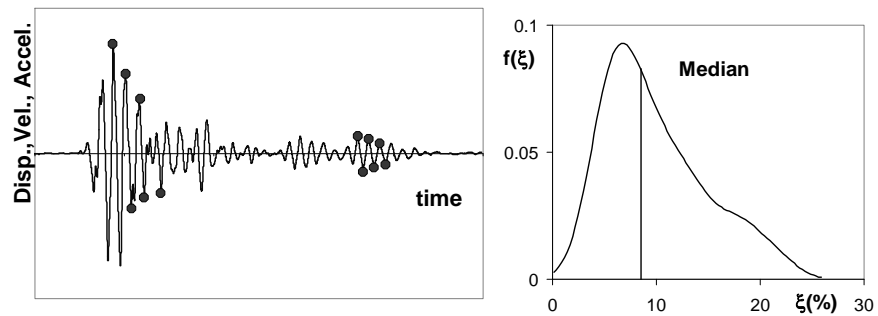
All frequencies can be evaluated by a Short Time Fourier Transform (STFT) [Gabor, 1946] applied on the whole signal, as shown in Figure 1, that refers to an earthquake recorded on a real building [Laurenzano et al. 2010, Mucciarelli et. al 2004]. This particular analysis allows to describe the variation in the time of the main frequencies of the structure.

The last instrumental parameter considered in the method is the variation of equivalent structural viscous damping  $\Delta \xi$  related to the first structural modes. Information about damping can enrich the quality and quantity of the knowledge on the global damage, particularly if such information is associated to the other parameters above described.

The damping can be estimated for non stationary signal using the *only output* non-parametric technique elaborated by Mucciarelli and Gallipoli [2007]. It measures the viscous equivalent damping of the signal recorded using a semi-probabilistic approach. The damping is estimated by the logarithmic decrement method on minimum of three consecutive decreasing peaks separated by the same period  $T$ , as shown in Figure 2a. The period  $T$  is assumed considering a fixed tolerance  $\varepsilon$  as function of  $T$ . The routine output supplies a vector  $\xi = (\xi_1, \xi_2, \dots, \xi_n)$ , where  $\xi_i$  is the value obtained by two consecutive decreasing peaks. Then, the lognormal distribution of the vector  $\xi$  is evaluated in order to get a median value of distribution for all signal duration, as shown in Figure 2b.



**Figure 1. Signal recorded and STFT of the signal.**

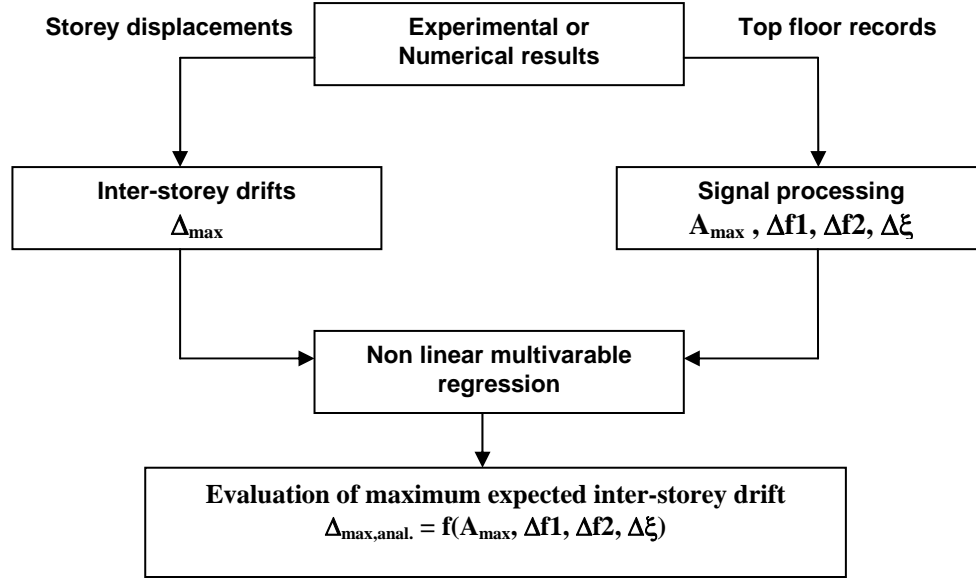


**Figure 2. (a) Logarithmic decrement method on signal recorded (b) Statistical distribution**

A check on the effectiveness of the result is, then, carried out by comparing the fit curve between theoretical and empirical cumulative distribution function, through the Kolmogorov-Smirnov test [Piccolo, 1998]. The variation  $\Delta\xi$  between the equivalent viscous damping evaluated on the signal before and after the earthquake is assumed as reference parameter in the method.

### 1.2 Regression Analysis for Structural Constant Evaluation

In order to verify the existence of a relationship between the selected parameters and the maximum inter-storey drift, several correlation analyses have been developed considering: (i) the outcomes of experimental shaking table tests carried out within the research projects TREMA and POP [Dolce et al. 2005, 2006]; (ii) numerical non linear dynamic analysis carried out considering several structure configurations and different natural accelerograms selected in the European Strong Motion Database and compatible with the actual European Seismic Code. The flow-chart shown in Figure 3 explains the approach.



**Figure 3. Scheme of approach used in the method.**

Considering the output of each experimental or numerical test, a matrix containing the maximum inter-storey drift (dependent parameter) along the height of the structure and the four instrumental parameters above defined (independent parameters) is built (eq. 3) in order to perform the correlation and regression analyses. Each row refers to a single test or numerical analysis.

$$\begin{matrix}
 \Delta_{\max 1} & a_{\max 1} & \Delta f_{11} & \Delta f_{21} & \Delta \xi_1 \\
 \Delta_{\max 2} & a_{\max 2} & \Delta f_{12} & \Delta f_{22} & \Delta \xi_2 \\
 \dots & \dots & \dots & \dots & \dots \\
 \Delta_{\max n} & a_{\max n} & \Delta f_{1n} & \Delta f_{2n} & \Delta \xi_n
 \end{matrix} \quad (3)$$

In the case of structural damage the global non linear behaviour of the building makes the linear approach ineffective and a non linear multivariable regression becomes necessary to describe the relationship between the considered parameters and the maximum drift. In this study a non linear polynomial relationship, with eight coefficients, has been considered to describe the correlation between the maximum drift and other parameters, as shown in the following paragraphs. The  $c_1, c_2, \dots, c_8$  constants are determined, for each single structure, by starting from all experimental or numerical data and solving the oversized equation system (4). They constitute the characteristic factors of the structure.

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta_n \end{pmatrix} = \begin{pmatrix} a_{\max 1}^2 & a_{\max 1} & \Delta f_{11}^2 & \Delta f_{11} & \Delta f_{21}^2 & \Delta f_{21} & \Delta \xi_1^2 & \Delta \xi_1 \\ a_{\max 2}^2 & a_{\max 2} & \Delta f_{12}^2 & \Delta f_{12} & \Delta f_{22}^2 & \Delta f_{22} & \Delta \xi_2^2 & \Delta \xi_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{\max n}^2 & a_{\max n} & \Delta f_{1n}^2 & \Delta f_{1n} & \Delta f_{2n}^2 & \Delta f_{2n} & \Delta \xi_n^2 & \Delta \xi_n \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{pmatrix} \quad (4)$$

Finally, the analytical expression to evaluate the maximum expected drift along the height of the building can be written as:

$$\Delta_{an} = c_1 \cdot a_{\max}^2 + c_2 \cdot a_{\max} + c_3 \cdot \Delta f_1^2 + c_4 \cdot \Delta f_1 + c_5 \cdot \Delta f_2^2 + c_6 \cdot \Delta f_2 + c_7 \cdot \Delta \xi^2 + c_8 \cdot \Delta \xi \quad (5)$$

The weight of each single instrumental parameters in the evaluation of the maximum analytical inter-storey drift is estimated by the following expression:

$$W_i = \frac{|F_i|}{\sum_i |F_i|} \quad (6)$$

## 2 EXPERIMENTAL TESTS

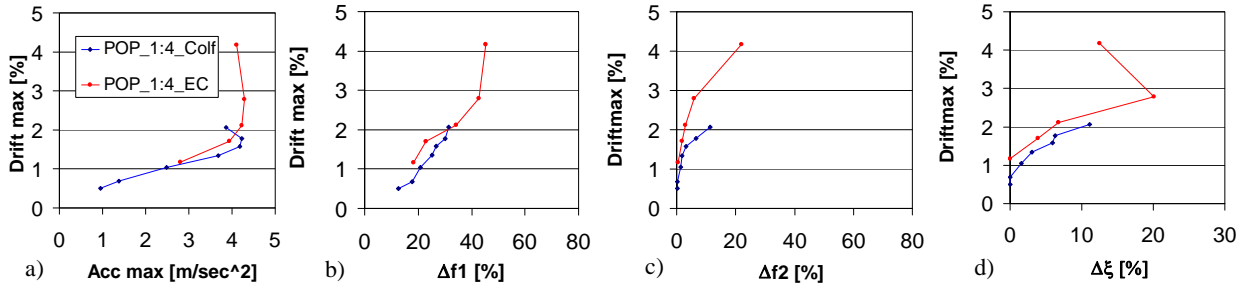
The outcomes of two extensive experimental programs consisting in dynamic tests on two similar 1/4 scaled 3-D R/C models, derived from a prototype building designed for gravity loads only, have been considered. The first 4-storey model was tested trough the uni-axial shaking table facility of the Structural Laboratory of the University of Basilicata in Potenza, within the POP project [Dolce et al., 2005]. The second model was a 3-storey specimen, with infill panels, tested on the 4x4m 6-DOF shaking table facility of ENEA Casaccia, (Rome) within the TREMA project [Dolce et al., 2006].



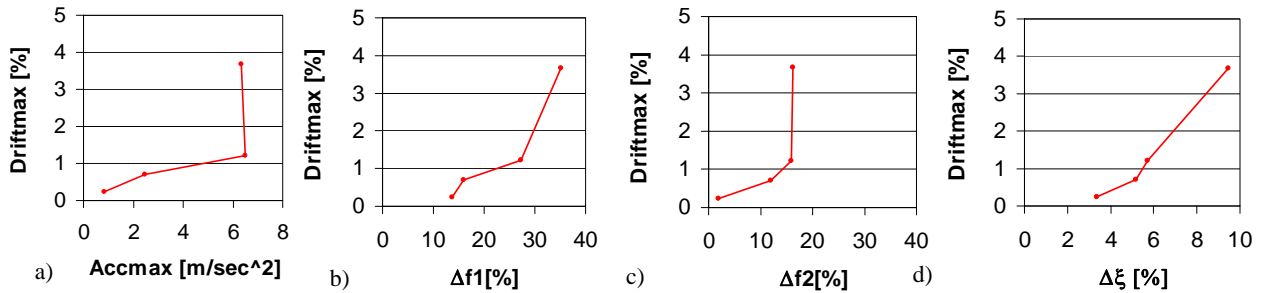
Figure 4. Experimental models a) POP project and b) TREMA project

The correlation analyses here presented were carried out considering, for the POP model, the structural response under both natural and artificial uni-directional earthquakes, while just natural bi-directional earthquake response was considered for the TREMA model. The natural seismic input employed in both projects was the Colfiorito record of the Italian 1997 Umbria-Marche earthquake, while the artificial seismic input considered was derived from the response spectrum provided by Eurocode 8 for soil type B. Both acceleration profiles were scaled down in time by a  $(4)^{1/2}$  factor, for consistency with the scale of the model. For the fixed-base configuration tests carried out within the POP Project the effective peak acceleration of the table was progressively increased from 0.05g up to 0.35g. All floor displacement were

measured through *Temposonic* digital transducers, fixed to an external steel reference frame. The floor accelerations were acquired through an horizontal servo accelerometers system. In the TREMA project tests, the effective peak acceleration of the table was progressively increased from 0.04g up to 0.23g. The accelerations of the top floor were acquired through 3 horizontal servo accelerometers and their measures were recomposed in the centre of mass. Figures 5 and 6 show the maximum experimental inter-storey drifts obtained for both POP and TREMA experimental tests vs each considered instrumental parameters. The maximum inter-storey drift  $\Delta_{max}$  is almost linearly dependent from the maximum top acceleration  $a_{max}$ , at least for values that do not involve significant damage for the structure, as shown in Figures 5a and 6a.



**Figure 5. POP. Correlation among maximum inter-storey drift and instrumental parameters.**



**Figure 6. TREMA. Correlation among maximum inter-storey drift and instrumental parameters.**

It is worth to note that higher values of acceleration, that would result in a damage of the structural elements, produce, a strong increasing of the drift. Indeed, other parameters  $\Delta f_1$ ,  $\Delta f_2$  and  $\Delta \xi$  (Fig. 5b,c,d and 6b,c,d) show a non linear dependence on the maximum inter-storey drift  $\Delta_{max}$ , more evident for  $\Delta \xi$  and  $\Delta f_2$ , and a dispersion degree considerably bigger than  $a_{max}$ . It is interesting to observe that both frequency variations  $\Delta f_1$  and damping variation  $\Delta \xi$  assume values different from zero for  $\Delta_{max}$  greater than about 0.4-0.5%. This value represents the threshold for structural damaging.

The analytical inter-storey drift evaluated by the non linear regression analysis, as summarised by equation 5, was compared to the experimental results. Then, for each specimen a single vector of characteristic parameters ( $c_1, \dots, c_8$ ) was determined starting from all experimental data relevant to examined structure. Figures 7 and 8 show the experimental vs analytical maximum inter-storey drifts. In this case the correlation factor between the two parameters ( $R^2 \approx 1$ ) demonstrates the good estimation of maximum drift provides by the proposed formulation. In Figures 9 and 10 the weight of each single instrumental parameter used in the non linear regression to estimate the inter-storey drift are showed. Maximum top acceleration  $a_{max}$  provides the most important contribution to  $\Delta_{max}$ , especially for low intensity earthquakes. In this case of the POP model the frequency variation  $\Delta f_1$  accounts for 20÷25%. For increasing seismic intensity, and therefore for damage cumulating, the contribution of the two previously parameters is getting down, to advantage of  $\Delta f_2$  and  $\Delta \xi$ . A different trend for TREMA model was observed, probably this difference comes from the presence of masonry infill and from damage occurred in previous tests. In any case the analytical formulation was able to correctly evaluate the maximum drift.

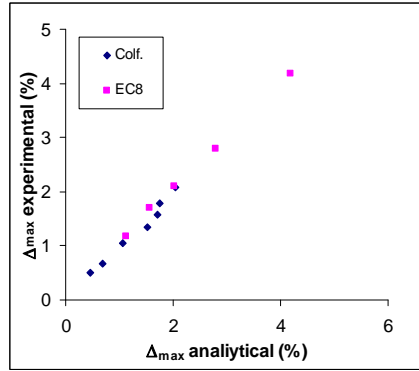


Figura 7: POP Model: experimental vs analytical maximum interstorey drift interstorey

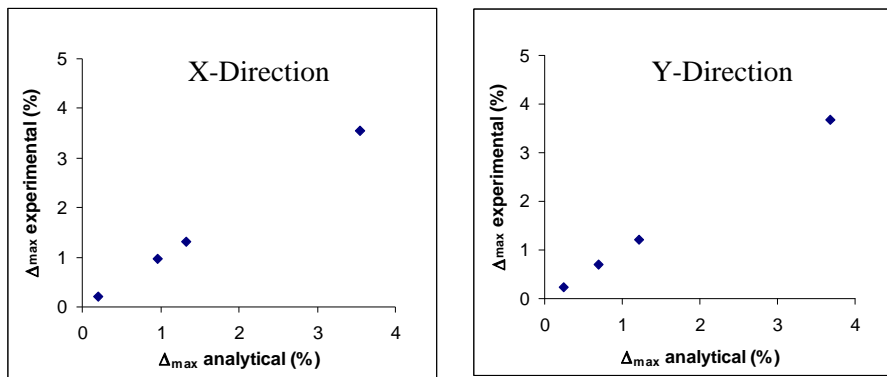


Figura 8: TREMA model: experimental vs analytical maximum interstorey drift

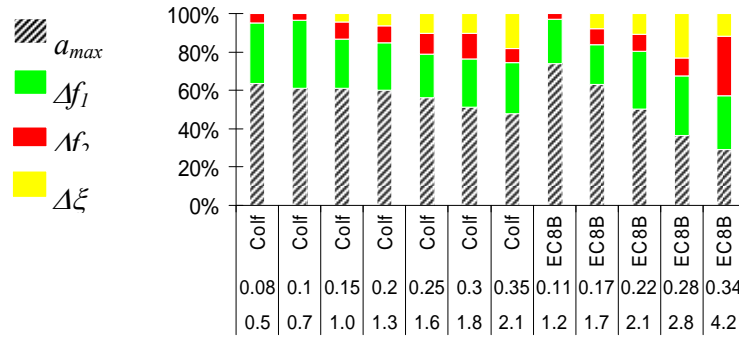


Figura 9: POP Model: weight contribute for each instrumental parameter vs experimental test.

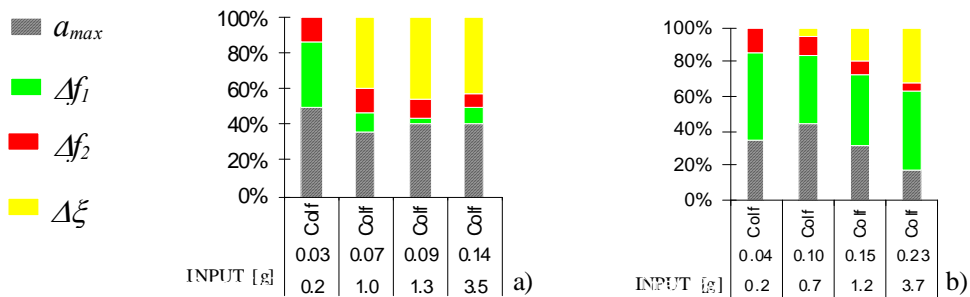


Figura 10: TREMA Model: weight contribute for each instrumental parameter vs experimental test

### 3 NUMERICAL SIMULATION

In order to understand the influence of the seismic input and of the most significant geometrical and mechanical characteristic of the monitored building on the effectiveness of the algorithm for damage detection, a parametric numerical simulation study has been planned. A huge amount of numerical data is currently being processed. The considered characteristics have been: storey's number, total building height, inter-storey height, different ratios among X and Y in plan dimensions, presence of infill panels, presence of soft storey and different codes and rules for the building design.

The preliminary results up now analysed and explained in this paragraph were obtained by considering a nonlinear numerical model of a R/C regular building, five floor, four frames along the longitudinal direction (X) and three frames in the transverse direction (Y) and having a rectangular plan, 15x12m.

The considered structure has been designed following the criteria of the actual Italian seismic code [OPCM 3431/2005] for high ductility class (CDA), high seismic intensity (PGA 0.35g) and for stiff soil type A. The inter-storey height is constant and equal to 3m, for a total building height equal to 15m. A software based on non linear finite element method (SAP2000 non-linear) [Wilson 2002] has been used to model the 3-D structure. Beams and columns have been modelled using frame elements, assuming 20 MPa cylindrical strength of concrete and 430 MPa yield strength of steel. In order to simulate a non linear structural behaviour during a strong ground motion, link elements and plastic hinges have been added at the end of beam and column elements respectively. Link elements have a Takeda hysteretic behaviour, while plastic hinges have an axial load-dependent behaviour. A total of 168 non-linear bi-directional dynamic analyses have been carried out using, for each peak ground (raising from  $a_g=0.05g$  up to  $0.35g$ ) and for each soil type considered (A and B,C,E -  $S=1, 1.25$ ), seven different couples of natural accelerograms Eurocodes compatible, selected from the European Strong Motion Database ([www.reluis.it](http://www.reluis.it)). For soft soil, type D, seven pairs of artificial accelerograms, having a spectrum compatible with Italian seismic code, have been considered. Figure 11 shows the response spectra for each group of seismic inputs.

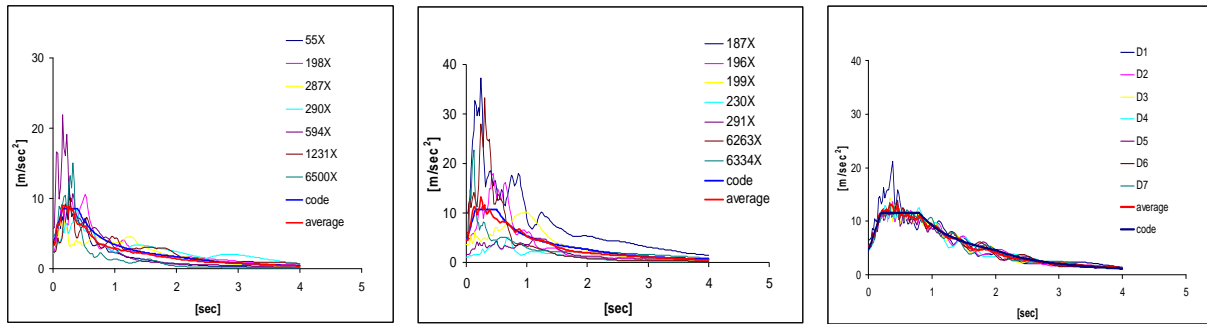


Figure 11: Response spectra of the numerical seismic input

Figure 12 shows the maximum numerical inter-storey drifts vs all the considered parameters. These parameters have been evaluated by elaborating the displacement and acceleration histories of the master joint at each floor and for all considered earthquakes. The maximum inter-storey drift  $\Delta_{max}$  (Fig. 12a) is still linear dependent from maximum top acceleration  $a_{max}$ . Moreover in the diagram of Figure 12a an evident low dispersion of the results can be observed. Other parameters  $\Delta f_1$ ,  $\Delta f_2$  and  $\Delta \xi$  (Figg. 12b,c and d) show a dispersion of results bigger than  $a_{max}$ , while frequency variation  $\Delta f_2$  and damping variation  $\Delta \xi$  confirm the same trend observed for experimental correlation analyses.

For the numerical model a regression analysis has been made considering the results of all numerical simulations. Therefore, a single set of characteristic constants ( $c_1, \dots, c_8$ ) for the considered structure has been carried out.



In Figure 13 the maximum inter-story drifts obtained from numerical analyses and those evaluated by the analytical procedure (eq. 5) for all considered cases are compared. As for experimental tests, also in this case the analytical procedure provides a good estimation of the drift values, as certified by a very good correlation coefficient ( $R^2=1$ ).

In Figure 14 the weight of each single parameter on the analytical evaluation of the maximum inter-story drift with equation 5 is reported. The trend observed with experimental results was confirmed by the non linear analyses outcomes. For low values of peak ground acceleration (PGA) and for all soil typologies, the maximum top acceleration  $a_{max}$  provides the more consisting contribution on  $\Delta_{max}$ , while for increasing of PGA values, the frequency variation  $\Delta f_2$  and  $\Delta \xi$  become more important.  $\Delta f_2$  maintain the same importance.

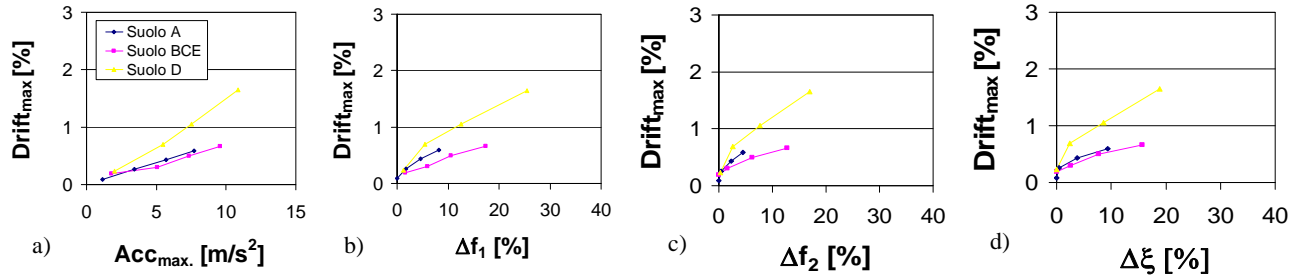


Figure 12. Correlation among maximum inter-story drift and instrumental parameters.

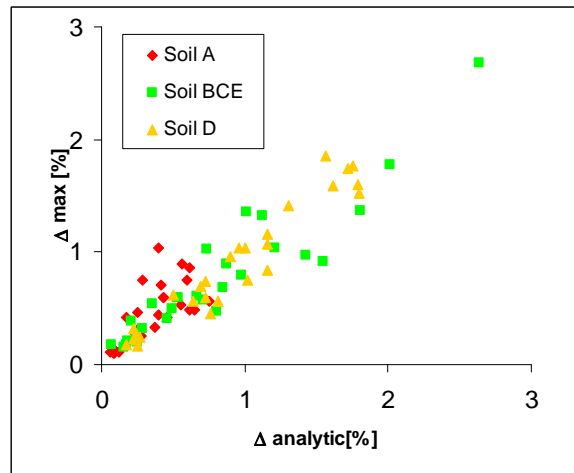


Figure 13. Experimental vs analytical maximum inter-story drift for all soil A-BCE-D.

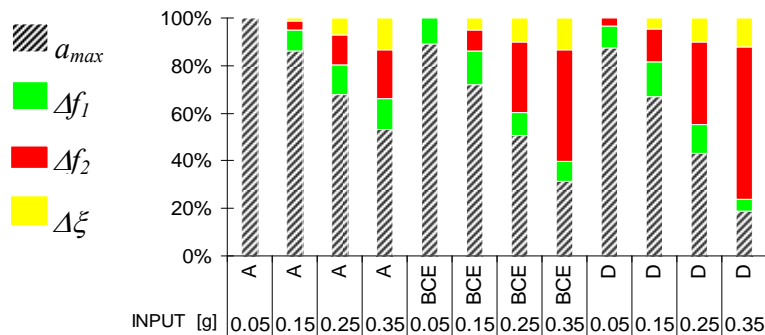


Figure 14. weight of each instrumental parameter in the max drift evaluation.

## 4 CONCLUSIONS

The damage assessment methodology proposed in this paper can detect and quantify the damage suffered by a R/C building using only few parameters evaluated starting from instrumental data recorded on the top of the monitored structure (Level I methods). The damage is expressed in terms of maximum inter-storey drift (damage indicator) evaluated by means of a non linear regression analysis, starting from parameters defined above. Several outcomes obtained from experimental and numerical tests allowed to calibrate and to verify the methodological approach. The proposed approach could result very useful especially for high excitation level and considering that it takes only a few instruments, it should favor the possibility to increase the number of monitored strategic structures. The first analyses of the results show that, for regular buildings, the only case investigated up to now, the maximum value of the top floor acceleration is the most strictly related parameter to the maximum inter-storey drift along the building height. The other parameters become important for high PGA values, when a strong non linear structural behaviour is activated.

## Acknowledgements

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