S-Transform: A Band-Variable Filter to Extract the Nonlinear Dynamic Behaviour of Soil and Structures

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ABSTRACT

The Fourier transform is certainly one of the main tools used to study the dynamic response of systems. This integral transformation is very useful and reliable if one wants to investigate the response of a stationary system, i.e. a system that doesn't changes its characteristics over time. On the contrary, when the study on the evolution of the dynamic response of a system whose characteristics vary with time is required, the Fourier transform is no longer reliable. In this regard, several mathematical tools to analyze time-variable dynamic responses were developed. Soil and buildings, subject to transient forcing such as an earthquake, may change their characteristics over time with initiation of non-linear phenomena. In most of cases, both for soil and buildings, abandoning the linear elastic field could represent a problem. This paper proposes a new methodology to approach the study of non-stationary response of soil and buildings: a band-variable filter based on the S-Transform. In fact, thanks to the possibility of change the bandwidth over time, it becomes possible to extract from a generic signal only the response of the system focusing on the variation of individual modes of vibration. In this paper, possible applications of the band variable filter to study the non-stationary response of soil and buildings have been proposed.

Keywords: *S-Transform, Band-Variable Filter, Structural Health Monitoring, Dynamic Identification, Nonlinear Dynamics*

1 INTRODUCTION

In the past years, several techniques for signal analysis and structural dynamic identification were proposed with the aim to evaluate the dynamic characteristics of soil and structures. One of the main tools used in the past years is the Fourier Transform and some tools derived from the same hypothesis such as Transfer Function and others techniques based on transformations from time to frequency domain. In recent years many authors have proposed techniques to overcome some of the limitations of the classical analyses when signals are non-stationary [1, 9]. For structural engineers, non-stationarity in the seismic signal recorded within a building is generally linked to the variability of the input (i.e. earthquake, explosion, etc..), to the non-linear behaviour of the structure (i.e. damage evolution), to the dynamic interaction between structure and soil or adjacent structures. The study of the evolution over time of dynamic characteristics of a system has often been addressed in an approximate way, using tools that require the assumption of stationarity, and these techniques might not be appropriate. For example, one of the main tools used to analyze dynamical systems is the Fourier transform. This technique, as well as all those tools which found their basis on the assumption of stationary behaviour of the system, appears to be not appropriate when studies on

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system that changes its characteristics over time. In order to overcome some of the limitations derived from the Fourier Transform, another tool widely used to study the transients is the STFT (Short Time Fourier Transform) [9]. This technique exceeds certain limits of the simple Fourier transform, giving some indications on the variation over time of the spectral characteristics of analyzed signal. Unfortunately, also this integral transformation has some limits which tend to distort the result. More recently, several techniques for time-frequency signals analysis have been developed. The most widely used are the Wavelet Transform [11] and the Wigner-Ville Distribution [2]. These other transformations provide a number of advantages compared with the Fourier transform and the STFT, but do not allow a fair assessment of the local spectrum. In other words, these instruments are insufficient to a correct evaluation of the spectral characteristics taking into account the instantaneous variations of the local spectrum. A tool that overcomes these limitations and allows to accurately assess both the spectral characteristics and their local variation over time is the S transform [16]. As it has been defined in the mathematical transformation, the properties of Gaussian window scale derive from the continuous wavelet [11]. However, in this case the constrain of zero mean for the wavelets it is not requested. Furthermore, compared to the wavelet transform, the S-Transform changes the shape of the real and imaginary coefficients over time together with the temporal translation of the Gaussian window. On the contrary, the wavelet transform does not have this property: the entire waveform translates over time, but never changes its shape [14]. The S-Transform also has many important properties, for example, the S-Transform can be written as an operator that is a function of the Fourier spectrum and it is a linear operator [16].Thanks to this property it is possible to extract the processed information from the signal of interest, without modifying the local spectra characteristics. Taking advantage of these properties, the S-Transform has been used up to now in seismology and earthquake engineering to clean signals from unwanted noise. Examples of the use of S-Transform to enhance the signal to noise ratio are given in [4, 6, 14]. The main goal of this paper is to provide a band-variable filter able to extract from a non-stationary signal only the phase of interest. It will be discussed the possibility on the use of the band-variable filter to extract from a signal recorded on a structure (excited by an earthquake) the response related to a single mode of vibration for which the related frequency changes over time if the structure is being damaged.

2 THE PROPOSED FILTER

This paper discusses the possibility to use the S-filter to extract the dynamic characteristics of systems that evolve over time by acting simultaneously in both frequency and time domain. The filter was built using the properties of convolution, linearity and invertibility of the S-Transform. As discussed in Ditommaso et al. [6], the proposed band-variable filter can be written as:

$$h_f(t) = \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} [S(\tau, f) * G(\tau, f)]\, d\tau \right) \cdot e^{i \cdot 2 \cdot \pi \cdot f \cdot t}\, df \qquad (1)$$

Where $S(\tau, f)$ is the S-Transform of the signal $h(t)$, $G(\tau, f)$ is the filtering matrix and $h_f(t)$ is the filtered signal. A generic k-th column $G(\tau_k, f)$ identifies a filtering window at a given time step. In order to better clarify the filtering procedure a worked example will be provided in the next section. Defining the filtering matrix could be a time-demanding operation, since each filtering window should be defined over a varying frequency recursively with a fixed time lapse. Moreover, the band-pass portion of the filter must be enlarged or reduced for each time step according to the desired bandwidth and the rectangular shape of the matrix. In order to simplify this process, it is convenient to try to automate it. We propose an implementation that defines the filtering matrix using a graphical user interface (GUI) that visualizes the amplitude spectra of the S-transform of the signal allowing for a point-and-click selection of the desired portion of the time-frequency domain.

The routine is designed so that the user can select a few points and the computer performs a cubic spline interpolation. In order to build the filtering matrix we used the *pimf* function of MATLAB©, which is a convenient approximation of a boxcar function. This spline-based curve is so named because of its Π shape. It is possible to obtain more details about this function from www.mathworks.com. It is possible to download the matlab routine related to the proposed Band Variable Filter from http://roccoditommaso.xoom.it.

3 APPLICATIONS

In the following we show two case histories that illustrate how it is possible to extract from real or synthetic recordings the information of interest: the first signal was recorded on the ground during an experimental campaign on a full-scale structure; the second one is related to the use of the band-variable filter to localize structural damage on RC structures. Particularly, the second application regards a nonlinear numerical RC model with infill panels subjected to earthquake.

3.1 Application 1: Soil-Structure Interaction

As discussed in Dolce et al. [8], the test structure of the ILVA-IDEM Project is an old R/C building, built in the 1970s in the former industrial area of Bagnoli (Naples). The building originally had two stories, one span in the transverse direction and twelve spans in the longitudinal direction. The interstory height is about 3.0 m for both the first and second stories. The span length is 5.60 m in the transverse direction, and varies between 2.80 and 3.80 m in the longitudinal direction.

(a)

(b)

(c)

Figure 1 – (a) the office building before the experiment, (b) contrast frame steel structure, (c) RC tested structure and the accelerometric station on the ground.

Two structural engineering groups took advantage of the availability of this building to test retrofitting measures. The group of the University of Basilicata upgraded one bay of the structure in the transverse direction with four recentering braces based on the superelastic properties of NiTi shape memory alloys (full details on this experiment are given by Dolce et al. [8]). As depicted in Figure 1 to avoid any interaction with the structural elements, all internal and external infill masonry panels were demolished; then the structure was subdivided into six similar modules. Using the accelerometric station located 10m far from the tested structure, the induced ground motion during a 7-cm displacement test were measured. This structural displacement is representative of the maximum excitation that this kind of building might withstand during an earthquake. A displacement of 7 cm over a total height of about 6.5 m is close to 1% interstory displacement index (IDI). Exceeding this value may lead to heavy damage to non-structural and structural elements of the building [15]. On the ground the signals were recorded using a sampling frequency equal to 100Hz. Previous work [4, 10] showed how the seismic waves generated by the experimental system had different dominant frequencies: the fundamental one is about 1-2 Hz due to the main structure, all the others are related to higher modes of the RC building and the eigenfrequencies of the steel contrast frame structure. It is worth noting that the fundamental frequency was variable due to the non-linear and dissipative effect of the retrofitting braces. Figure 2 shows the displacements recorded in correspondence of the accelerometric station placed 10m away from the building and both the S-Transform of the registration in the frequency range 0-35 Hz.

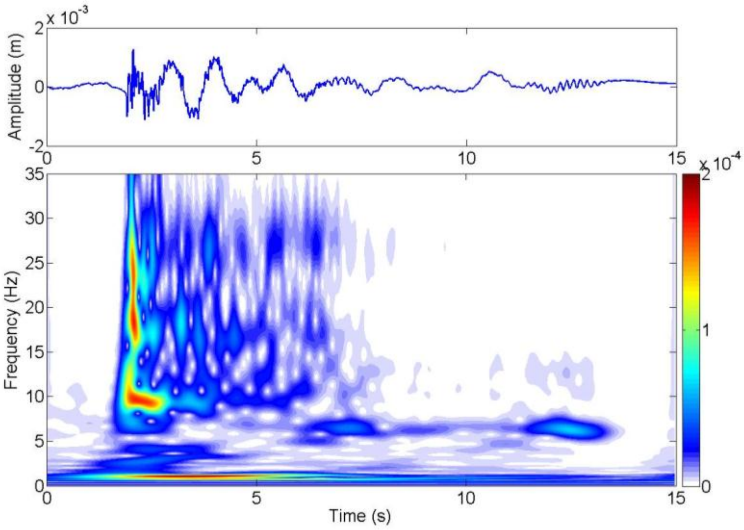


Figure 2 – Displacement Recorded 10m Far From The RC Building During The Release Test And The Related S-Transform.

Here we are interested to show how it is possible to use the principle of superposition to retrieve the main part of the recorded displacement time-histories (Figure 2) by using the band variable filter. Particularly, we are interested to demonstrate that most of the energy contained in the signal recorded at the station located 10m far from the RC structure was released by oscillating buildings and it is possible to separate the different components by mean the proposed filter.

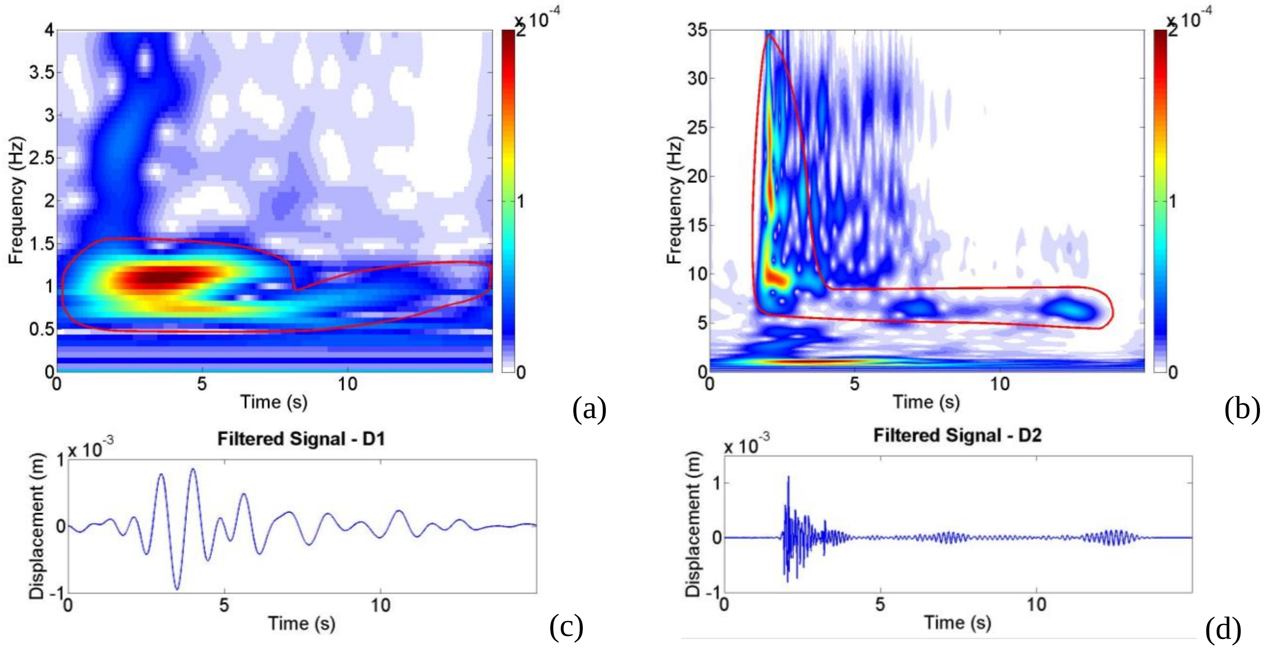
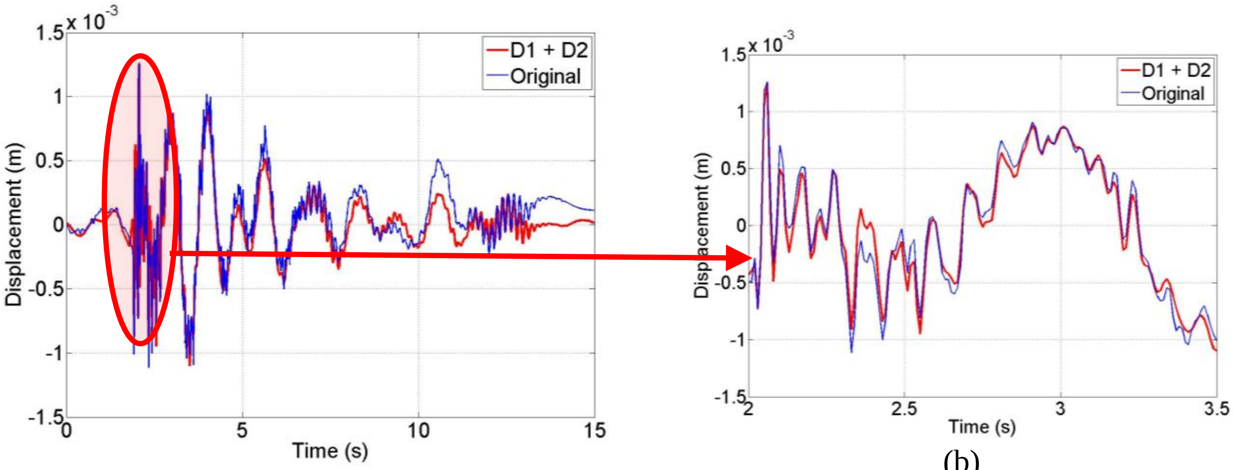


Figure 3 – Filtered Signals.

In the first step the component related to the fundamental mode of the RC structure was retrieved from the displacement time-histories using the band variable filter as depicted in Figure 3a and 3c (Signal D1). In the second step using the band variable filter the energy content related to higher modes of the RC structure and the modes related the contrast frame structure were been selected (Figure 3b) and retrieved in the time domain (Figure 3d). In Figure 4, a comparison between the original signal (blue line) and the signal obtained from the superposition of the signals (D1 and D2) depicted in Figures 3c and 3d (red line) was done.

It is worth noting that using the band variable filter in the time-frequency domain it is possible to divide the different contributions in the recorded non-stationary signal and, after that, it is possible to sum the retrieved signals in the time-domain (D1 and D2) to obtain a single signal composed only by the selected phases. It is important to highlight the possibility to preserve the original phase angles by using the proposed nonlinear filter. The superposition of original and band-variable filtered signals (Figure 4a) shows that both amplitude and phase are correctly preserved by the forward and inverse transformation provided by the S-Transform (Figure 4b).



(a) (b)

Figure 4 – (a) Comparison Between Original and Sum Of D1 and D2 signals (Filtered Signals); (b) Zoom of the time-window 2-3.5s

3.2 Application 1: Structural Damage Location

In this section, using a numerical nonlinear model built using SAP2000 finite elements program [3], we are interested to discuss about the possibility to use the proposed band variable filter to evaluate the geometric characteristics of structural mode shapes during the strong motion phase [6]. In fact, as shown in the previous paragraphs using the S-Transform it is possible to estimate the dynamic characteristics of a generic system with a good resolution both in time and frequency domain. Practically, using the proposed band variable filter, it is possible to extract the mode shapes of the structure step by step also during the phase of maximum nonlinearity. A nonlinear numerical model of an R/C regular building (Figure 5a), five floors, four frames along the longitudinal direction (X) and three frames in the transverse direction (Y), having a rectangular plan, 15×12m, has been considered. The considered structure has been designed following the criteria of the Italian seismic code (OPCM 3431/2005) for high ductility class (CDA), high seismic intensity (PGA 0.35g) and for soft soil type D. The height of each storey is 3m, for a total height of the building equal to 15m.

$$b_w = 0.2 \cdot d_w \cdot \sin(2\vartheta) \cdot \left(\frac{E_w \cdot t_w \cdot h_w^3 \cdot \sin(2\vartheta)}{E_c \cdot I_p}\right)^{-0.1} \cong 0.1 \cdot d_w \qquad \text{(eq. 3.1)}$$

In order to take into account the presence of infill panels within the structural R/C frames and their interaction with the columns, both the masonry strength and stiffness contribution have been considered [8] by inserting two equivalent structural elements in the models. The mechanical characteristics of these elements were evaluated considering the Mainstone model [12] through the eq. 3.1. This relationship is valid for rectangular shape panels only. In the simulation a 12+8cm thick panel was considered. The Force-Displacement behaviour for Mainstone model is depicted in Figure 5b. Using SAP2000 finite elements program, these elements were modelled by mean multilinear plastic link. In order to consider the presence of doors and windows, the effective area of the infill panels considered in the analyses was reduced at 80% of the total area. Beams and columns have been modelled with frame elements, assuming 20MPa cylindrical strength of concrete and 430MPa yield strength of steel. In order to simulate a structural nonlinear behaviour during a strong ground motion, link elements and plastic hinges have been used at the end of beam and column elements respectively. Link elements have a Takeda hysteretic behaviour, while plastic hinges have an axial load-dependent one. It is possible to find more details about this numerical model in Ponzo et al. [15].

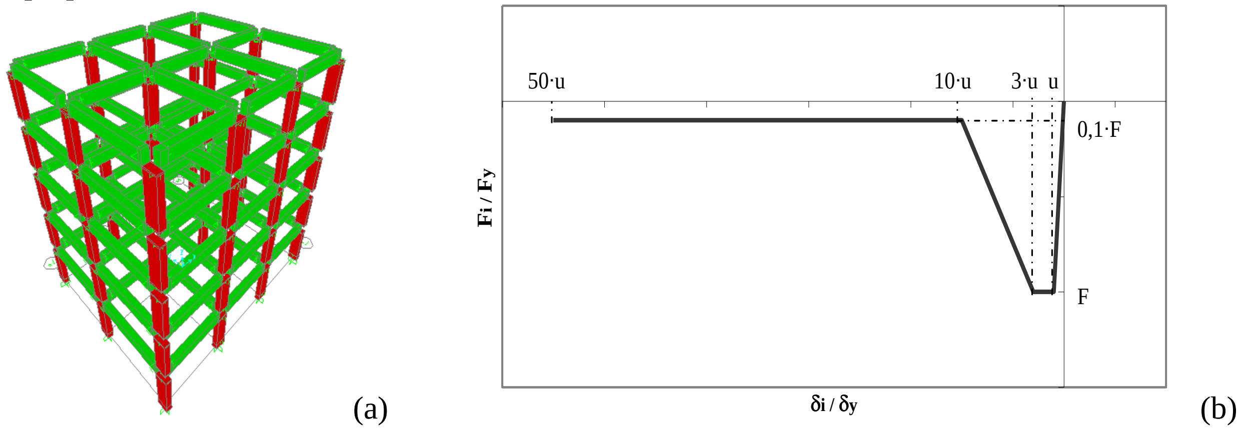


Figure 5 – (a) Numerical Model; (b) Displacement-Force Behaviour For Infill Panel Elements

With reference to the nonlinear dynamic behaviour, mode shapes related to the fundamental mode of vibration were extracted using the signals filtered by mean the band variable filter following these steps:

- evaluation of structural response at the last floor where the fundamental mode is very clear;
- creation of the filtering matrix based on the S-Transform from the accelerometric recording;
- convolution of the filtering matrix with signal recorded at each level;
- evaluation of the mode shape over time.

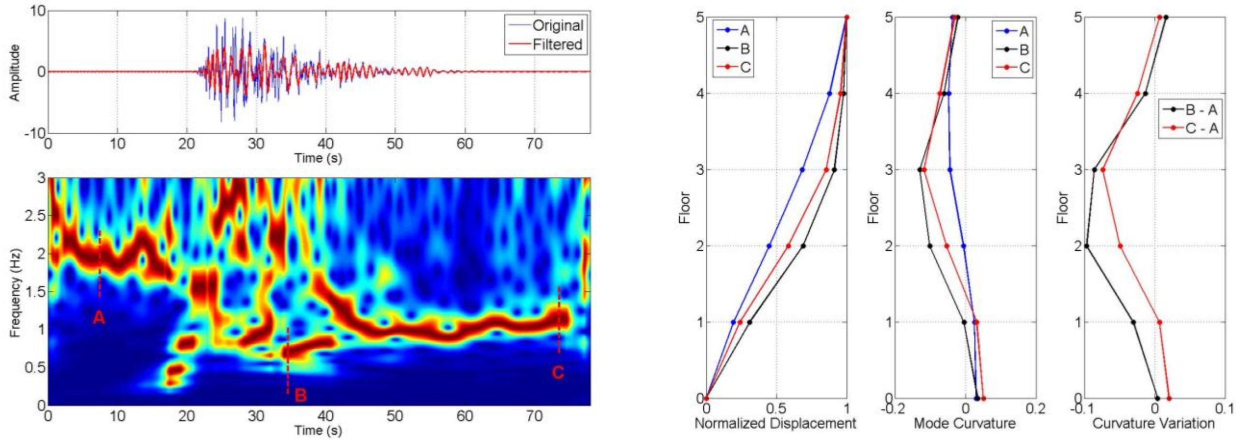


Figure 6 – (a) Normalized S-Transform; (b) From The Left: Mode Shapes And Curvatures evaluated at time instances A, B and C, And Curvature Variations B-A And C-A

Figure 6a shows a comparison between the original signal (blue) recorded on the top of the building (along the fundamental mode direction) and the signal filtered using the procedure described above. Also the normalized S-Transform of the signal is depicted. The behaviour of the structure is clearly non-stationary, with a change over time of the frequency of the fundamental mode. The starting frequency is 2.0Hz, the minimum frequency is 0.65Hz and the final frequency is 1.15Hz. Figure 6a shows the frequency evolution of the fundamental mode of the structure extracted using the band variable filter and the time-point from which the mode shapes were evaluated: Mode Shapes A, B and C (starting frequency, minimum frequency and final frequency). It is worth noting that using the standard approach it would have been possible to evaluate only the starting and final mode shapes, on the contrary, using the band variable filter it is possible to evaluate also the mode shape related to the minimum frequency recorded during the maximum excursion in the plastic field. Figure 6b shows the mode shapes evaluated over time as indicates in Figure 6a: Mode Shape B has a curvature bigger than Mode Shapes A and C. It is known that mode curvature is strongly linked to the building damage during a seismic event [13]. Therefore, being able to evaluate the mode curvature during the maximum excursion in nonlinear field and isolating it from superimposed signals, we can achieve a better understanding of the mechanisms of damage as well as a more precise location of the damage on the structure.

4 CONCLUSIONS

Dealing with signal analysis and dynamic characterization of structures means a constant search of mathematical tools that enable to study in detail the phase of non-stationary response of dynamic structures. Techniques based on Fourier transform provide good results when the response of the system is stationary, but fail when the system exhibits a non-stationary, time-varying behaviour. To hamper classical techniques, it is not necessary that a building reaches damage: even the non-stationarity of the input and the possible interaction with the ground and/or adjacent structures can show the inadequacy of classic techniques [7]. In 1996, Stockwell introduced a new powerful tool for the signals analysis: the S-Transform. Compared with the classical techniques for the time-frequency analysis, this transformation shows a much better resolution and also offers a range of fundamental properties such as linearity and invertibility [6]. By exploiting these properties, it was possible to develop a filter whose band varies both in time and in frequency domains, being very useful to study the characteristics of non-stationary signals. As discussed in the previous sections, this tool becomes necessary when one wants to isolate the response of individual, time-varying modes of vibration of soil and buildings (e.g., when their dynamic characteristics evolve over time as a result of seismic events). The ability to investigate the non-stationary response of soils and buildings opens new scenarios, giving the opportunity to explore new possibilities. For example, the ability to isolate individual modes of vibration of a building make possible to explore their variation over time, evaluating the change in mode curvature. It is known that this parameter is strongly linked to the building damage during a seismic event [13]. Therefore, being able to evaluate the mode curvature during the maximum excursion in nonlinear field and isolating it from superimposed signals, allows for a better understanding of the mechanisms of damage as well as for a more precise location of both structural and non-structural damage.

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REFERENCES

[1] Addison, P. S. (2002). The illustrated wavelet transform handbook: introductory theory and applications in science, engineering, medicine and finance. *IOP Publishing.* ISBN 0750306920.

[2] Bradford S Case, Yang Jing, Heaton Thomas (2006). Variations in the dynamic properties of structures: the Wigner-Ville distribution. *Proceedings of the 8th U.S. National Conference on Earthquake Engineering.* April 18-22, 2006, San Francisco, California, USA.

[3] Computers & Structures, Inc. "SAP2000 – Integrated Software for Structural Analysis & Design, *Technical Reference Manual*". (www. http://www.csiberkeley.com/).

[4] Ditommaso, R., M. Mucciarelli, M. R. Gallipoli and F. C. Ponzo (2010a). Effect of a single vibrating building on free-field ground motion: numerical and experimental evidences. *B EARTHQ ENG*, 8(3) 693-703. DOI: 10.1007/s10518-009-9134-5.

[5] Ditommaso, R., S. Parolai, M. Mucciarelli, S. Eggert, M. Sobiesiak and J. Zschau (2010b). Monitoring the response and the back-radiated energy of a building subjected to ambient vibration and impulsive action: the Falkenhof Tower (Potsdam, Germany). *B EARTHQ ENG*, 8(3) 705-722. DOI: 10.1007/s10518-009-9151-4.

[6] Ditommaso R., Mucciarelli M., Ponzo F. C. (2012a). Analysis of non-stationary structural systems by using a band-variable filter. *B EARTHQ ENG.* DOI: 10.1007/s10518-012-9338-y.

[7] Ditommaso R., Mucciarelli M., Parolai S., Picozzi M. (2012b). Monitoring the structural dynamic response of a masonry tower: comparing classical and time-frequency analyses. *B EARTHQ ENG.* DOI: 10.1007/s10518-012-9347-x.

[8] Dolce, M., D. Cardone, R. Marnetto, M. Mucciarelli, D. Nigro, C. F. Ponzo, and G. Santarsiero (2004). Experimental static and dynamic response of a real r/c frame upgraded with SMA recentering and dissipating braces. *Proc. 13th World Conf. on Earthquake Eng.*, Vancouver, British Columbia, Canada, 1–6 August 2004, paper no. 2878, CDRom Edition.

[9] Gabor, D. (1946). Theory of communications. *J. Inst. Electr. Eng*, Vol. 93, pp. 429–457.

[10] Gallipoli, M. R., M. Mucciarelli, F. Ponzo, M. Dolce, E. D'Alema, and M. Maistrello (2006). Buildings as a Seismic Source: Analysis of a Release Test at Bagnoli, Italy. *B SEISMOL SOC AM*, 96(6) 2457–2464.

[11] Mallat, S. (1998). A Wavelet Tour of Signal Processing. *Academic*, New York.

[12] Mainstone R.J. (1974). Supplementary note on the stiffness and strength of infilled frames. Current paper CP13/74. *Build. Res. Establishment*. London.

[13] Pandey, A. K., M. Biswas, M. M. Samman (1991). Damage detection from changes in curvature mode shapes. *Journal of Sound and Vibration*, 145(2) 321-332.

[14] Parolai, S. (2009). Denoising of Seismograms Using the S Transform. *B SEISMOL SOC AM*, 99(1) 226–234.

[15] Ponzo F. C., Ditommaso R., Auletta G., Mossucca A. (2010). A Fast Method for Structural Health Monitoring of Italian Strategic Reinforced Concrete Buildings. *B EARTHQ ENG*, 8(6) 1421-1434. DOI: 10.1007/s10518-010-9194-6.

[16] Stockwell, R. G., L. Mansinha, and R. P. Lowe (1996). Localization of the complex spectrum: the S transform. *IEEE Trans. Signal Process.*, 44 998–1001.